Quantum phase transitions beyond the Landau paradigm in a Sp(4) spin system

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We propose quantum phase transitions beyond the Landau's paradigm of Sp(4) spin Heisenberg models on the triangular and square lattices motivated by the exact $Sp(4) \simeq SO(5)$ symmetry of spin-3/2 fermionic cold atomic system with only *s*-wave scattering. On the triangular lattice, we study a phase transition between the $\sqrt{3} \times \sqrt{3}$ spin ordered phase and a Z_2 spin liquid phase; this phase transition is described by an O(8) sigma model in terms of fractionalized spinon fields, with significant anomalous scaling dimensions of spin order parameters. On the square lattice, we propose a deconfined critical point between the Neel order and the valence bond solid (VBS) order, which is described by the CP(3) model, and the monopole effect of the compact U(1) gauge field is expected to be suppressed at the critical point.

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I. INTRODUCTION

Landau's classic phase-transition paradigm describes continuous phase transitions by symmetry breaking of the system[1] and the powerful renormalization-group theory, developed by Wilson, suffices this paradigm with systematic calculation techniques. Based on Landau-Ginzburg-Wilson (LGW) theory,[2] the continuous phase transition should be described by fluctuations of physical order parameters. A few years ago, it was proposed that a direct unfine-tuned continuous transition between two-ordered phases, which break different symmetries is possible in quantum magnet[3,4] (which is forbidden in Landau's theory). Recent numerical results suggest that this transition may exist in a SU(2) spin-1/2 model with both Heisenberg and ring exchange.[5,6] The key feature of this non-Landau critical behavior is that at the critical point the field theory in terms of fractionalized objects with no obvious physical probe is a more appropriate description. In spite of the difficulty of probing the fractionalized excitations, the fractionalized nature of the critical point leads to enormous anomalous dimension of the physical order parameter that is distinct from the Wilson-Fisher fixed-point or the mean-field result, which can be checked experimentally.

In a seminal paper, it was proved that in spin-3/2 cold atom systems, with the standard *s*-wave scattering approximation, the four-component spin-3/2 fermion multiplet enjoys an enlarged $Sp(4) \simeq SO(5)$ symmetry without fine tuning any parameter.[7] By tuning the spin-0 and spin-2 scattering channels, there is one point with an even larger $SU(4) \supset Sp(4)$ symmetry.[7–9] The fundamental representation of the 15 generators of SU(4) Lie algebra can be divided into two groups. Γ_a with $a=1,2\cdots5$ and $\Gamma_{ab}=\frac{1}{2i}[\Gamma^a,\Gamma^b]$. Γ_a obey the Clifford algebra $\{\Gamma^a,\Gamma^b\}=2\delta_{ab}$. Let us denote the fermion atom operator as ψ_α, then the fermion bilinear $\hat{\Gamma}_a=\psi^\dagger\Gamma_a\psi$ form a vector representation of Sp(4) group and $\hat{\Gamma}_{ab}=\psi^\dagger\Gamma_{ab}\psi$ form an adjoint representation of Sp(4) group. In the particular representation, we choose,

$$\Gamma_a=\sigma^a\otimes\mu^z,\quad a=1,2,3,\quad \Gamma^4=1\otimes\mu^x,\quad \Gamma^5=1\otimes\mu^y. \tag{1}$$

The difference between SU(4) algebra and Sp(4) algebra is that two Sp(4) particles can form a Sp(4) singlet through a 4×4 antisymmetric matrix $\mathcal{J}=i\sigma^y\otimes\mu^x$, which satisfies the following algebra:

$$\mathcal{J}^t=-\mathcal{J},\mathcal{J}^2=-1,\mathcal{J}\Gamma_{ab}\mathcal{J}=\Gamma^t_{ab},\mathcal{J}\Gamma_a\mathcal{J}=-\Gamma^t_a. \tag{2}$$

One can see that $\mathcal{J}_{\alpha\beta}\psi^\dagger_\alpha\psi^\dagger_\beta$ creates a Sp(4) invariant state, therefore, the valence bond solid (VBS) state of SU(2) spin systems can be naturally generalized to Sp(4) spin systems. By contrast, two SU(4) particles can only form a six-dimensional representation and a ten-dimensional representation of SU(4) algebra and the smallest SU(4) singlet always involves four particles.

If we consider a Mott-Insulator phase of spin-3/2 cold atoms on the optical lattice with one particle per well on average, the effective spin Hamiltonian should be invariant under Sp(4) transformations. The most general Sp(4)-Heisenberg model contains two terms:

$$H=\sum_{\langle i,j\rangle}J_1\hat{\Gamma}^{ab}_i\hat{\Gamma}^{ab}_j-J_2\hat{\Gamma}^a_i\hat{\Gamma}^a_j. \tag{3}$$

The key difference between $\hat{\Gamma}_{ab}$ and $\hat{\Gamma}_a$ is their behavior under time-reversal transformation. The time-reversal transformation on the fermion multiplet ψ_α is $\psi_\alpha\to\mathcal{J}_{\alpha\beta}\psi_\beta$; this implies that $\hat{\Gamma}_{ab}$ ($\hat{\Gamma}_a$) is odd (even) under time reversal. Also, if rewritten in terms of the original SU(2) spin-3/2 matrices, Γ_{ab} only involves the odd powers of spins and Γ_a only involves the even powers of spins.[9] This model can be exactly realized in spin-3/2 cold atom systems, the coefficients J_1 and J_2 are determined by the spin-0 and spin-2 scattering parameters.[9] Clearly when $-J_2=J_1$, the system has SU(4) symmetry. In this work we will consider the Heisenberg model on the triangular and square lattice, in the parameter regime with $J_1>0$. Our focus in the current work will be the non-Landau-type quantum phase transitions, which is also a larger spin generalization of the deconfined criticality discussed before. A more detailed analysis of the whole phase diagram of the Sp(4) Heisenberg model in Eq. (3) will be given in a future work.[10]

II. Sp(4) HEISENBERG MODEL ON THE TRIANGULAR LATTICE

Let us study the triangular lattice first, then we will use the standard Schwinger boson formalism to study the magnetic ordered phase. We introduce Schwinger boson spinon b_α, as usual $\hat{S}_i^a = b_{i,\alpha}^\dagger S_{\alpha\beta}^a b_{i,\beta}$, where $\hat{S}^a$ are the 15 generators of SU(4) algebra in the fundamental representation. This definition of spinon b_α is subject to a local constraint: $\Sigma_{\alpha=1}^4 b_{i,\alpha}^\dagger b_{i,\alpha}=1$, which also manifests itself as a local U(1) degree of freedom:$b_{i,\alpha} \to \exp(i\theta_i) b_{i,\alpha}$. Using the following identities:[11]

$$\Gamma_{\alpha\beta}^{ab}\Gamma_{\gamma\sigma}^{ab} = 2\delta_{\alpha\sigma}\delta_{\beta\gamma} - 2\mathcal{J}_{\alpha\gamma}\mathcal{J}_{\beta\sigma},$$

$$\Gamma_{\alpha\beta}^{a}\Gamma_{\gamma\sigma}^{a} = 2\delta_{\alpha\sigma}\delta_{\beta\gamma} + 2\mathcal{J}_{\alpha\gamma}\mathcal{J}_{\beta\sigma} - \delta_{\alpha\beta}\delta_{\gamma\sigma}, \tag{4}$$

the Hamiltonian (3) can be rewritten as

$$H = \sum_{\langle i,j\rangle} 2(J_1 - J_2)\hat{K}_{ij}^\dagger \hat{K}_{ij} - 2(J_1 + J_2)\hat{Q}_{ij}^\dagger \hat{Q}_{ij},$$

$$\hat{K}_{ij} = b_{i,\alpha}^\dagger b_{j,\alpha}, \hat{Q}_{ij} = \mathcal{J}_{\alpha\beta} b_{i,\alpha} b_{j,\beta}. \tag{5}$$

Now we introduce two variational parameters $K_{ij}=\langle \hat{K}_{ij}\rangle$ and $Q_{ij}=\langle \hat{Q}_{ij}\rangle$ and assuming that these variational parameters are uniform on the whole lattice, the mean-field Hamiltonian for Eq. (3) reads,

$$H_{mf} = \sum_{\langle i,j\rangle} 2(J_1 - J_2)K\hat{K}_{ij} - 2(J_1 + J_2)Q\hat{Q}_{ij} + \text{H.c.}$$

$$- 2(J_1 - J_2)K^2 + 2(J_1 + J_2)Q^2 - \mu(b_{i,\alpha}^\dagger b_{i,\alpha} - 1). \tag{6}$$

The following formalism is similar to Ref. 12, which studied the SU(2) spin models on the triangular lattice. The term involving μ imposes the constraint on the Hilbert space of spinon $\Sigma_{\alpha=1}^4 b_{i,\alpha}^\dagger b_{i,\alpha}=1$. If the spectrum of the spinons is gapless, the spinon will condense at the minima of the Brillouin zone. By solving the self-consistent equations for K, Q, and μ, we obtain that when $J_2/J_1 > -0.3$, there is a finite percentage of spinon condensate at momenta $\pm\vec{q}_0 = \pm(2\pi/3, 2\pi/\sqrt{3})$, which are the corners of the Brillouin Zone. The condensate density as a function of J_2/J_1 is plotted in Fig. 1.

The gauge-field fluctuation rooted in the constraint $\Sigma_{\alpha=1}^4 b_{i,\alpha}^\dagger b_{i,\alpha}=1$ is the most important correction to the mean-field calculation above. The local constraint would in general induce U(1) gauge fluctuations. However, the condensate obtained from the Schwinger boson formalism corresponds to the state with nonzero expectation value $Q=\langle \hat{Q}_{ij}\rangle$, which is a pairing amplitude. The pairing between nearest-neighbor sites breaks the U(1) gauge symmetry down to Z_2 gauge symmetry, therefore, the long-wavelength field theory of this condensate should only have Z_2 gauge symmetry. To understand this order, we define slow mode z_α as

$$b_\alpha(x) = e^{i\vec{q}_0\cdot\vec{x}} z_\alpha(x) + e^{-i\vec{q}_0\cdot\vec{x}} \mathcal{J}_{\alpha\beta} z_\beta^*(x). \tag{7}$$

Now one can rewrite spin operators $\hat{\Gamma}_{ab}$ and $\hat{\Gamma}_a$ in terms of slow mode z_α as

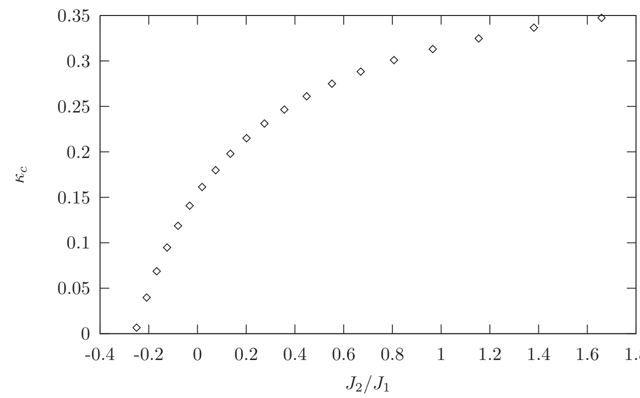

FIG. 1. Mean-field solutions on triangular lattice at different J_2/J_1 with $J_1>0$. The y axes κ_c show the density of spinon condensate, which is also proportional to $|z|^2$ as defined in Eq. (7). κ_c decreases to nearly zero (0.007) as J_2/J_1 decreases.

$$\hat{\Gamma}_{ab} \sim e^{i2\vec{q}_0\cdot\vec{x}} z\mathcal{J}\Gamma_{ab} z + \text{H.c.},$$

$$\hat{\Gamma}_a \sim z^\dagger \Gamma_a z = n_a. \tag{8}$$

Therefore $\hat{\Gamma}_a$ has a uniform order n_a, while $\hat{\Gamma}_{ab}$ is only ordered at finite momentum $\pm 2\vec{q}_0$. For completeness, one can define Sp(4) adjoint vector $n_{1,ab}$ and $n_{2,ab}$ as

$$n_{1,ab} = \text{Re}[z\mathcal{J}\Gamma_{ab}z], n_{2,ab} = \text{Im}[z\mathcal{J}\Gamma_{ab}z]. \tag{9}$$

The order of $\hat{\Gamma}_{ab}$ can be written in terms of $n_{1,ab}$ and $n_{2,ab}$,

$$\hat{\Gamma}_{ab} \sim \cos(2\vec{q}_0\cdot\vec{x}) n_{1,ab} + \sin(2\vec{q}_0\cdot\vec{x}) n_{2,ab},$$

$$\sum_{a,b} n_{1,ab} n_{2,ab} = 0. \tag{10}$$

$n_{1,ab}$ and $n_{2,ab}$ are two Sp(4) adjoint vectors "perpendicular" to each other. Since $\hat{\Gamma}_a$ is time-reversal even, while $\hat{\Gamma}_{ab}$ is time-reversal odd,[9] the condensate of z_α has both uniform spin nematic order and $\sqrt{3}\times\sqrt{3}$ order.

The U(1) local gauge degree of freedom is lost in Eq. (7). The residual gauge symmetry is only Z_2, which transforms $z \to -z$. Physically this implies that an arbitrary U(1) transformation of z field will result in a rotation of spin order parameter $\hat{\Gamma}_{ab}$. This situation is very similar to the spinon description of the $\sqrt{3}\times\sqrt{3}$ order of SU(2) spins on the triangular lattice.[13] The field theory describing this condensate should contain Z_2 gauge field. However, since Z_2 gauge field does not introduce any long-range interaction or critical behavior, we can safely integrate out the Z_2 gauge field. The field theory can then be written as

$$L = |\partial_\mu z|^2 + r|z|^2 + g(|z|^2)^2 + \cdots \tag{11}$$

The ellipses include all the Sp(4) invariant terms.

Apparently, without the ellipses, the Lagrangian (11) enjoys an enlarged O(8) symmetry once we define the real boson field multiplet $\vec{\phi}$ as $\vec{\phi}=(\text{Re}[z_1],\text{Im}[z_1],\cdots,\text{Im}[z_4])^t$ and the Lagrangian (11) can be rewritten as

$$L=\sum_{\alpha=1}^{8}(\partial_\mu\phi_\alpha)^2+r|\vec{\phi}|^2+g(|\vec{\phi}|^2)^2+\cdots \qquad (12)$$

The Lagrangian (12), without other perturbations, describes an O(8) transition and the ordered state has a ground-state manifold (GSM),

$$\mathrm{U}(4)/[\mathrm{U}(3)\otimes Z_2]=S^7/Z_2=\mathrm{RP}(7), \qquad (13)$$

we mod Z_2 from S^7 because of the Z_2 gauge symmetry of z. There are certainly other terms in the field theory, which can break the O(8) symmetry down to Sp(4) symmetry, but all the terms allowed by Sp(4) symmetry and lattice symmetry include at least two derivatives, for instance $|\mathcal{J}_{\alpha\beta}z_\alpha\partial_\mu z_\beta|^2$. These terms change the Goldstone mode dispersion but do not change the GSM, and since they contain high powers of z and also at least two derivatives, they are irrelevant at the O(8) critical point. Other Sp(4) invariant terms without derivatives such as $\Sigma_{a,b}(n_{1,ab})^2$, $\Sigma_a(n_a)^2$, $\epsilon_{abcde}n_{1,ab}n_{1,cd}n_e$, etc., either vanish or can be rewritten in terms of powers of $z^\dagger z$, which preserves the O(8) symmetry. Therefore, we conclude that the ground-state manifold of the condensate is S^7/Z_2 and the transition between the condensate and disordered state by tuning J_1/J_2 belongs to the O(8) universality class. This transition is beyond the Landau's paradigm in the sense that the field theories (11) and (12) are written in terms of spinon field instead of physical order parameters. The physical order parameters are bilinears of spinon, which implies that the anomalous dimension of the physical order parameters are enormous at this transition.

Since the GSM is S^7/Z_2 with fundamental group $\pi_1[S^7/Z_2]=Z_2$, in the condensate there are gapped visons, which is a "π-flux" of the "Higgsed" Z_2 gauge field. The disordered phase is actually a Z_2 spin liquid with gapped but mobile visons. This Z_2 spin liquid phase of SU(2) spin systems can be most conveniently visualized in the quantum dimer model (QDM) on the triangular lattice,[14] which by tuning the dimer flipping and the dimer potential energy, stabilizes a gapped phase with Z_2 topological order and with no symmetry breaking. As we discussed earlier, two Sp(4) particles can form a Sp(4) singlet, therefore, the QDM for Sp(4) spin systems is exactly the same as the SU(2) spins, with also a stable Z_2 spin liquid phase. Because the Z_2 spin liquid is a deconfined phase, the excitations of the Z_2 spin liquid include gapped Sp(4) bosonic spinons besides the visons. If we start with the disordered Z_2 spin liquid state and drive a transition by condensing the gapped Sp(4) spinon, the field theory of this transition is in the same form as Eq. (11).

Since on one site there is only one particle, the particular QDM is subject to the local constraint with one dimer connected to each site. This type of QDM is called odd QDM, since the product of Z_2 electric field around each site is $\Pi\sigma^x=-1$, which will attach a π-flux to each hexagon of the dual honeycomb lattice of the triangular lattice. This π-flux that is seen by the visons will lead to four degenerate minima in the vison band, and the condensation of the vison at these minima breaks the translation and rotation symmetry of the lattice,[15] and the transition has been suggested to be an O(4) transition.

In this section, we discussed the transition between the Z_2 spin liquid and the $\sqrt{3}\times\sqrt{3}$ state of the Sp(4) spin system. For comparison, let us briefly discuss the order-disorder transition of the $\sqrt{3}\times\sqrt{3}$ state in the standard Landau theory, ignoring the topological nature of the Z_2 spin liquid. In the $\sqrt{3}\times\sqrt{3}$ order, both time-reversal and Sp(4) spin symmetries are broken. A general Ginzburg-Landau-Lagrangian can be written in terms of the time-reversal even O(5) vector n^a, which is defined as the long-wavelength field of $\hat{\Gamma}^a$ and two adjoint vectors n_1^{ab} and n_2^{ab}, introduced in Eq. (10). At the quadratic level, none of these three vectors mix, while at the cubic order, a mixing term is allowed by the Sp(4) symmetry, $\Sigma_{i=1}^2\epsilon_{abcde}n_i^{ab}n_i^{cd}n^e$. This term implies that the ordering of the adjoint vectors would drive the order of n^a, but the statement is not necessarily true conversely. If the O(5) vector n_a is ordered while the adjoint vectors n_1^{ab} and n_2^{ab} are disordered, the system breaks the Sp(4) symmetry while preserving the time-reversal symmetry. Therefore, if the system is tuned toward the disordered phase, the Landau's theory allows for multiple transitions with the time-reversal symmetry restored before the Sp(4) symmetry. A uniform collinear order $\hat{\Gamma}_a$ has GSM $S^4=\mathrm{SO}(5)/\mathrm{SO}(4)$, therefore, the transition of n^a belongs to the O(5) universality class. The transition associated with time-reversal symmetry breaking is described by the O(10) vectors n_1^{ab} and n_2^{ab}, with various anisotropies in the background of the gapless O(5) ordering n^a. For instance, at the quartic order, there is a term which imposes the "orthogonality" between the two O(10) vectors $(\Sigma_{a,b}n_1^{ab}n_2^{ab})^2$. The nature of this transition requires more detailed analysis. By contrast, the Schwinger boson and field theory analysis show that there can be a direct O(8) transition between the phase with coexistence of n_a and $n_{i,ab}$ and a spin disordered phase with Z_2 topological order.

III. Sp(4) HEISENBERG MODEL ON THE SQUARE LATTICE

Now let us switch the gear to the square lattice. On the square lattice, at the point with $J_1=J_2>0$, the model [Eq. (3)] can be mapped to the SU(4) Heisenberg model with fundamental representation on one sublattice and conjugate representation on the other.[9] The equivalence can be shown by performing transformation $S^a\rightarrow\mathcal{J}^\dagger S^a\mathcal{J}$ on one of the sublattices and by using the identities in Eq. (2). At this point, $J_1=J_2$ has been thoroughly studied by means of large-N generalization[16–18] and quantum Monte Carlo.[19] It is agreed that at this point the spinon b_α condenses, and there is a small Neel moment. In the Schwinger boson language, the Neel state on the square lattice corresponds to the condensate of Schwinger bosons with nonzero expectation of $\langle\hat{Q}_{ij}\rangle$, which seems to break the U(1) gauge symmetry down to Z_2. However, the U(1) gauge symmetry can be restored if the Schwinger bosons on the two sublattices are associated with opposite gauge charges, therefore, the connection between spinon b_α and low energy field z_α is

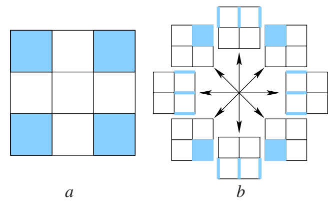

FIG. 2. (Color online) (a) The SU(4) plaquette order pattern, with four SU(4) particles around the colored squares, form a SU(4) singlet. (b) The particular type of VBS state depends on the phase angle of the monopole operator.

$$b_\alpha \sim z_\alpha, (\text{sublattice } A),$$

$$b_\alpha \sim \mathcal{J}_{\alpha\beta} z^\dagger_\beta, (\text{sublattice } B). \tag{14}$$

The GSM of the Schwinger boson condensate is

$$\mathrm{U}(4)/[\mathrm{U}(1)\otimes\mathrm{U}(3)] = S^7/\mathrm{U}(1) = \mathrm{CP}(3). \tag{15}$$

The field theory for this condensate is most appropriately described by the CP(3) model

$$L = \sum_{\alpha=1}^{4} |(\partial_\mu - i a_\mu) z_\alpha|^2 + r|z|^2 + g(|z|^2)^2 + \cdots \tag{16}$$

Again, if we perturb this field theory with Sp(4) invariant terms, the GSM is still CP(3) and the critical behavior is unchanged. The condensate of z_α has staggered spin order $\hat{\Gamma}_{ab}$ but uniform nematic order $\hat{\Gamma}_a$ on the square lattice.

In the condensate of z, gauge field a_μ is Higgsed; if z is disordered, a_μ would be in a gapless photon phase if the gauge fluxes are conserved. However, because $\pi_2[\mathrm{CP}(3)] = Z$, the ground-state manifold can have singular objects in the 2+1 dimensional space time,[16] which corresponds to the monopole of the compact U(1) gauge field a_μ. The conservation of gauge fluxes is broken by the monopoles, which due to its Berry phase will drive the system to a phase breaking the lattice symmetry.[16,20]

At another point with $-J_2 = J_1 > 0$, this model is SU(4) invariant with fundamental representations on both sublattices. This point is not so well studied. A fermionic mean-field theory[21] and an exact diagonalization[22] on a 4×4 lattice has been applied to this point. The results suggest that the ground state may be a plaquette order, as depicted in Fig. 2, with four particles forming a SU(4) singlet on every one out of four unit squares. A similar plaquette ordered phase is obtained on the spin ladder.[23] It is interesting to consider the dynamics of the plaquettes, for instance, in three–dimensional (3D) cubic lattice, a quantum plaquette model as a generalization of the quantum dimer model has been studied both numerically[24] and analytically.[25] If we perturb away from the SU(4) point with Sp(4) invariant terms, this plaquette order is expected to persist into a finite region in the phase diagram due to its gapped nature. This phase presumably can be continuously connected to the Sp(4) VBS state with Sp(4) singlets resonating on every one of four unit squares (Fig. 2) because both states are gapped and they

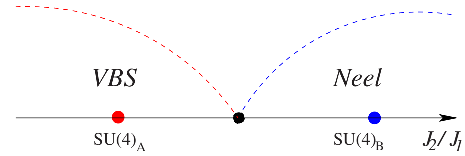

FIG. 3. (Color online) The conjectured phase diagram for the Heisenberg model in Eq. (3). By tuning one parameter J_2/J_1, the system evolves from the Neel state to the dimerized VBS state and the transition can be continuous. The dashed lines denote the magnitude of the Neel and VBS order parameter. $\mathrm{SU}(4)_\mathrm{A}$ is the SU(4) invariant point with fundamental representation on all sites and $\mathrm{SU}(4)_\mathrm{B}$ point is another SU(4) point with fundamental representation on one sublattice and conjugate representation on the other.

break the same lattice symmetry. More details about the possible phases on the square lattice is under study by another group.[26]

The dimer resonating plaquette state can be understood in the same way as the dimer columnar state and as the proliferation of monopoles of the compact U(1) gauge field then the oscillating Berry phase of the monopoles will choose the specific lattice symmetry-breaking pattern. Both the dimer columnar order and the dimer plaquette order can be viewed as a condensate of fluxes of U(1) gauge field with the U(1) conservation of fluxes breaking down to Z_4, and if the phase angle of the condensate is $2n\pi/4$, the system is in the columnar state. While if the phase angle is $(2n+1)\pi/4$, the system is in the dimer plaquette phase[16] (Fig. 2). If one considers a pure QDM on the square lattice, then the crystalline pattern can be obtained from the dual rotor model with the Lagrangian,[27,28]

$$L_d = (\partial_\mu \chi)^2 - \alpha \cos(8\pi\chi). \tag{17}$$

Here $\exp(i2\pi\chi)$ is the monopole operator, which creates a 2π flux of the U(1) gauge field. Now whether the system favors the columnar order or the dimer resonating plaquette order, it simply depends on the sign of α.

Now we conjecture a phase diagram (Fig. 3). Suppose J_1 is fixed and we tune J_2, if $J_2 > J_c$, then the system remains in the condensate of z, which is the Neel order of spin operators. When $J_2 < J_c$, then the system loses the Neel order and enters the VBS state. This transition can be a direct second-order transition and the field theory is described by the CP(3) model in Eq. (16), assuming the CP(3) model itself has a second-order transition. The most important instability on this field theory is the monopole of the compact U(1) gauge field, which is certainly relevant in the crystalline phase. However, it has been shown convincingly that at the 3D XY transition, the Z_4 anisotropy of the XY variable is irrelevant[29] and it was also argued that a large number of flavor of boson field tend to suppress the monopole effects.[4] Therefore, it is likely that the monopoles are irrelevant at the critical point described by field theory (16). Compared with the SU(2) spin system, our Sp(4) system with doubled number of complex boson fields has a better chance to ensure the irrelevance of monopole perturbations at the transition. We also want to point out that between the Neel order and the dimer resonating plaquette order, an intermediate phase with columnar order is also possible. But the transition between the Neel order and the columnar order is also described by field theory (16) and the columnar order is connected to the resonating plaquette order through a first-order transition.

IV. SUMMARY AND EXTENSION

In this work, we studied the quantum phase transitions beyond the Landau's paradigm in the spin-3/2 cold atom systems with emergent enlarged Sp(4) symmetry. Compared with the $J-Q$ model studied before,[5,6] the spin model we considered is very realistic; we propose that these results are observable in real experimental systems in the future. It would also be interesting to study the Heisenberg model in this work through numerical techniques. A careful numerical study of the classical CP(3) model without monopoles is also desired, as has been done recently for the SU(2) invariant CP(1) model.[30]

The current work focused on the parameter regime with $J_1>0$. In the regime with$J_1<0$, the Schwinger boson formalism would lead to the ordered state with nonzero expectation value $K=\langle\hat{K}_{ij}\rangle$ and the Schwinger bosons condense at momentum (0,0). This state is the ferromagnetic state with uniform order n^{ab} and n^{a}. The ferromagnetic state and the Neel state can be connected through a first-order transition. More theoretical tools are desired to determine the other parts of the phase diagram accurately. We will leave this to the future work.[10]

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